of this kind are demonstrably small. In particular, the plane $\psi=0$ is known to better than $\pm 5°$ from the mechanical measurements and from the distribution of dihedral planes for those events which did *not* pass through the magnetic field (Fig. 2, curve B). The result obtained from the above events is in disagreement with that obtained in the other experimental run at the Cosmotron by Cool *et al.*[6] They obtained a result of -1.5 ± 0.5 nm. More accurate experiments are clearly indicated.

[6] R. L. Cool, E. W. Jenkins, T. F. Kycia, D. A. Hill, L. Marshall, and R. A. Schluter, Phys. Rev. **127**, 1952 (1962).

ACKNOWLEDGMENTS

The authors gratefully acknowledge the assistance of R. L. Kuskowski in the preparation and performance of this experiment. They would also like to thank W. J. Evans, H. C. Hollis, and F. S. Onesto for their assistance with the experimental apparatus and A. Georgoulakis, T. D. Graunke, R. L. Blowney (Argonne), and P. Lee (University of Illinois) for their untiring efforts in scanning. We would like to thank Dr. R. H. Dalitz and Dr. K. Tanaka for informative discussions, and express our gratitude to the Brookhaven National Laboratory and the Cosmotron staff for their hospitality and support of this experiment.

PHYSICAL REVIEW VOLUME 129, NUMBER 2 15 JANUARY 1963

Low-Energy Pion-Pion Scattering. II*

Louis A. P. Balázs
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 23 August 1962)

A self-consistent calculation of low-energy π-π scattering is made in which, in addition to the usual P-wave resonance, an $I=0$, D-wave resonance is also retained. The only free parameter is the pion mass. The resulting resonances have masses of 685 and 892 MeV, respectively, and the half-width is about 160 MeV in each case. The procedure consists of combining the Chew-Mandelstam and generalized Ball-Wong techniques with self-consistency. An outline is also given of a possible generalization of such a procedure to an arbitrary S-matrix process.

INTRODUCTION

A METHOD for making calculations in the low-energy π-π problem was given in an earlier paper,[1] henceforth referred to as I. The nearby singularities were treated by the conventional Chew-Mandelstam approach,[2] while the more distant ones were taken into account by a generalization of the Ball-Wong technique.[3] These two techniques were then combined with the requirement of self-consistency. An approximate calculation was made in which we consistently neglected everything except the P wave. Such a calculation, in which the only free parameter is the pion mass, can give us a self-sustaining resonance.

Recently, however, it has been conjectured, on the basis of the Regge-pole hypothesis, that there is also present an $I=0$, D-wave resonance in the π-π problem, with a mass of about 1 BeV.[4] Such a resonance will be shown to arise even if we have only the P-wave resonance of I in the crossed channel, although the mass is then too small. However, a coupled P-D calculation, in which both the P- and D-wave resonances are consistently retained, gives masses roughly consistent with the expected values. These calculations, of course, are all made in the elastic approximation. To increase the accuracy of the calculation without adding phenomenological information would require some method for calculating inelastic processes. In the final section, a generalization of the method given in I to such processes is outlined.

* This work done under the auspices of the U. S. Atomic Energy Commission.

[1] L. A. P. Balázs, Phys. Rev. **128**, 1939 (1962).

[2] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

[3] J. S. Ball and D. Y. Wong, Phys. Rev. Letters **6**, 29 (1961).

[4] This is where the topmost $I=0$ Regge trajectory passes through Re$l=2$. See G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **7**, 394 (1961); *ibid.* **8**, 41 (1962). (The author is indebted to Professor G. F. Chew for pointing out the possible importance of this resonance.)

THE P- AND D-WAVE RESONANCES

In I, the partial-wave amplitude for orbital angular momentum l and isotopic spin I was given by

$$A_{(l)I}(\nu)=N_l^I(\nu)/D_l^I(\nu), \tag{1}$$

with

$$D_l^I(\nu)=1-\frac{\nu-\nu_0}{\pi}\int_0^\infty d\nu'\left(\frac{\nu'}{\nu'+1}\right)^{1/2}\frac{R_l^I(\nu')N_l^I(\nu')}{(\nu'-\nu_0)(\nu'-\nu)}, \tag{2}$$

and

$$N_l^I(\nu)=A_{(l)I}(\nu_0)+\frac{\nu-\nu_0}{\pi}\int_{\nu_L}^{-1}d\nu'\frac{\mathrm{Im}A_{(l)I}(\nu')D_l^I(\nu')}{(\nu'-\nu_0)(\nu'-\nu)}+(\nu-\nu_0)\sum_{i=1}^{n}\frac{F_{(l)I}{}^i}{\omega_i+\nu}, \tag{3}$$

where $\nu=(s/4)-1$ if the pion mass$=1$, ν_0 is some subtraction point, and s is the square of the total energy in the barycentric system. The last term was obtained by making the approximation

$$\frac{1}{1+x\nu}=\sum_{i=1}^{n}\frac{G_i(x)}{1+x_i\nu}\qquad(4)$$

for $0<x<-\nu_L^{-1}$, where $\nu_L>-9$ and $x_i=\omega_i^{-1}$. The function $R_l^I(\nu)$ is the ratio of total partial-wave cross section to elastic partial-wave cross section, and is unity in the elastic approximation, which we shall use throughout. We can determine the constants $A_{(l)I}(\nu_0)$ and $F_{(l)I}^i$ by imposing two conditions: (a) that $A_{(l)I}(\nu)\propto\nu^l$ for small ν, and (b) that Eqs. (1), (2), and (3) give the same value and the same first $(n-l)$ derivatives of $A_{(l)I}(\nu)$ as given by

$$A_{(l)I}(\nu)=\frac{4}{\pi\nu}\int_0^\infty d\nu'\times\mathrm{Im}\tilde{A}_I\left(\nu',1+2\frac{\nu+1}{\nu'}\right)Q_l\left(1+\frac{\nu'+1}{\nu}\right)\qquad(5)$$

at some point ν_F in the region $\nu_L<\nu<0$. To find $\mathrm{Im}\tilde{A}_I$, we can always expand in the crossed channel

$$\mathrm{Im}\tilde{A}_I\left(\nu',1+2\frac{\nu+1}{\nu'}\right)=\sum_{I'=0}^{2}\beta_{II'}\sum_{l'=0}^{\infty}(2l'+1)\times\mathrm{Im}A_{(l')I'}(\nu')P_{l'}\left(1+2\frac{\nu+1}{\nu'}\right),\qquad(6)$$

where

$$\beta_{II'}=\begin{bmatrix}1/3 & 1 & 5/3\\ 1/3 & 1/2 & -5/6\\ 1/3 & -1/2 & 1/6\end{bmatrix}.$$

Equation (5) can also be used to obtain the discontinuity across the nearby part of the left-hand cut. This leads to Eq. (IV-7) of Chew and Mandelstam[2] or Eq. (5) of I. In particular, if we retain only a zero-width resonance at ν_R in the crossed channel, the left-hand cut starts at $\nu=-\nu_R-1$. By taking $\nu_L=-\nu_R-1$, we therefore eliminate the integral in Eq. (3).

To set up such a zero-width approximation, we shall generalize the procedure followed in I. If we first put $\mathrm{Re}D_l^I(\nu)\approx(\nu-\nu_R)/(\nu_0-\nu_R)$ and $-[\nu/(\nu+1)]^{-1/2}\times\mathrm{Im}D_l^I(\nu)=N_l^I(\nu)\approx(\nu/\nu_R)^lN_l^I(\nu_R)$, we obtain

$$\mathrm{Im}A_{(l)I}(\nu)=\frac{\nu^l(\Gamma_l^I)^2[\nu^{2l+1}/(\nu+1)]^{1/2}}{(\nu-\nu_R)^2+(\Gamma_l^I)^2[\nu^{2l+1}/(\nu+1)]}.\qquad(7)$$

where

$$\mathrm{Re}D_l^I(\nu_R)=0,\qquad(8)$$

and

$$\nu_R^l\Gamma_l^I=(\nu_R-\nu_0)N_l^I(\nu_R).\qquad(9)$$

Taking the zero-width limit, we obtain

$$\mathrm{Im}A_{(l)I}(\nu)=\pi\nu_R^l\Gamma_l^I\delta(\nu-\nu_R).\qquad(10)$$

In the P-wave approximation in I, a straight-line interpolation was used in Eq. (4). In other words, we set $n=2$, and put

$$G_{1,2}(x)=(x-x_{2,1})/(x_{1,2}-x_{2,1}),\qquad(11)$$

where $x_1=0.16$ and $x_2=0.02$; i.e., $\omega_1=6.25$ and $\omega_2=50$. Taking $\nu_0=\nu_F=-2$, and inserting Eqs. (10) and (6) into Eq. (5) with $l'=1$, we calculated $A_{(1)1}(\nu_0)$, $F_{(1)1}^1$, and $F_{(1)1}^2$ in the manner described above. These, in turn, were used to calculate ν_R and Γ_1^1 by means of Eqs. (2), (3), (8), and (9). It was then required that these calculated values equal the assumed ones. This gave $\nu_R\Gamma_1^1=2.6$ and $\nu_R=3.4$ (i.e., the mass $m_R=585$ MeV). A plot of the cross section

$$\sigma_l^I=4\pi(2l+1)[\nu(\nu+1)]^{-1/2}\,\mathrm{Im}A_{(l)I}(\nu)\qquad(12)$$

has a half-width of 110 MeV if we use Eq. (7) and 125 MeV if we use Eqs. (1), (2), and (3). The difference is small and justifies the use of the simpler form given by Eq. (7).

In the above calculation we implicitly assume that the results are not sensitive to changes in the ω_i consistent with the approximation (4). To test this assumption, we make large changes in ω_1 and ω_2, and repeat the calculation. With $\omega_1=6.25$ and $\omega_2=100$, we obtain $\nu_R\Gamma_1^1=2.2$ and $\nu_R=3.5$, while with $\omega_1=10$ and $\omega_2=50$ we have $\nu_R\Gamma_1^1=2.9$ and $\nu_R=4.1$ ($m_R=630$ MeV). Thus, the resonance parameters, and particularly the mass, are not very sensitive to changes in the values of ω_1 and ω_2, even if the changes make the approximation (4) marginal.

To calculate the $I=0$, D-wave resonance, we shall use the specific approximation made in Sec. 4 of a previous paper,[5] hereafter called S; i.e., we set $n=3$ and put

$$G_{1,2}(x)=\left(\frac{x-x_{2,1}}{x_{1,2}-x_{2,1}}\right)-\theta(x_2-x_1)\left(\frac{x-x_2}{x_3-x_2}\right)^2\left(\frac{x_3-x_{2,1}}{x_{1,2}-x_{2,1}}\right),\qquad(13)$$

and

$$G_3(x)=[(x-x_2)/(x_3-x_2)]^2\theta(x_2-x),\qquad(14)$$

where θ is the usual step function, $x_1=0.17$, $x_2=0.07$, and $x_3=0.012$. That this is a valid approximation is evident from Fig. 1(b) of S if we replace y by $5x$ and $20q^2$ by ν. If we retain only a P-wave resonance with $\nu_R\Gamma_1^1=2.6$ and $\nu_R=3.4$ in the crossed channel, and calculate the $I=0$, $l=2$ state by the above method, we obtain a resonance with $\nu_R^2\Gamma_2^0=0.8$ and $\nu_R=3.5$. The corresponding mass is comparable with the one for the P wave, contrary to expectations. However, the very existence of such a resonance shows that this state cannot be neglected in a self-consistent calculation.

To increase the accuracy of this result, a coupled P-D calculation was therefore made, in which both the P-

[5] L. A. P. Balázs, Phys. Rev. 125, 2179 (1962).

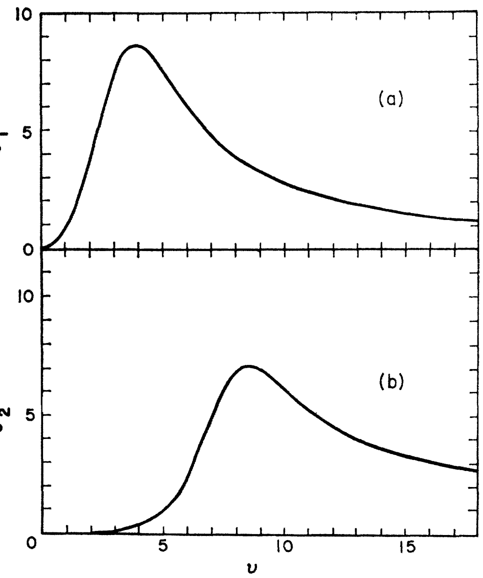

FIG. 1. Plots of σ_l^I as given by Eqs. (12) and (7) for: (a) the P-wave resonance with $\nu_R\Gamma_1^1=4.6$ and $\nu_R=5.0$; (b) the D-wave resonance with $\nu_R^2\Gamma_2^0=4.4$ and $\nu_R=9.2$.

and D-wave resonances were retained in the zero-width approximation in the crossed channel. With the same x_i, this calculation gave $\nu_R\Gamma_1^1=4.6$ and $\nu_R=5.0$ ($m_R=685$ MeV) for the P wave, and $\nu_R^2\Gamma_2^0=4.4$ and $\nu_R=9.2$ ($m_R=892$ MeV) for the D wave. The latter mass is compatible with the expected value of about 1 BeV, while the former is consistent with the experimental values of 725 to 770 MeV.[6,7] A plot of Eq. (12),using Eq. (7), on the other hand, gives a half-width of about 160 MeV in each case. For the P wave, this is several times the values 50 to 75 MeV deduced from experiment.[6,7] The discrepancy is probably caused by a combination of the crude approximations of our present work, and by the inadequacy of the simple models normally used for extracting experimental π-π cross sections from pion-production experiments.

INELASTIC EFFECTS AND ARBITRARY S-MATRIX PROCESSES

In the preceding calculations, inelastic effects were not included explicitly. This does not mean that such effects are completely neglected, since it was shown in I that they are partly taken into account if the approximation (4) is valid also for negative x. However, this presupposes that inelastic scattering is not yet too important at the energies of interest, which is already only a crude assumption in the case of the D-wave resonance. To increase the accuracy of the calculation, we must explicitly insert inelastic effects. This, in turn, requires a general method for handling any S-matrix process.

Consider such a process with an arbitrary number of incoming and outgoing particles, for which the square of the total energy is s. If we follow the Landau-Cutkosky rules,[8,9] we must consider all possible reduced graphs, which will consist of direct and exchange graphs (see Fig. 2). Direct graphs are those that have s-variable discontinuities which can be directly calculated by the Cutkosky generalized unitarity condition.[9] The remaining we call exchange graphs. If we project out a particular partial-wave amplitude, we will then have a function of s with the usual left- and right-hand cuts arising from the exchange and direct graphs, respectively. These cuts may, of course, include complex singularities and be overlapping.

Now, at a fixed value of s, the total amplitude may be written as a sum of integrals over discontinuities arising from exchange graphs, if the amplitude is considered as a function of some variable t_1 which is independent of s. This, in turn, gives the full partial-wave amplitude in the nearby left-hand region if we project out a particular wave. In particular, we can then find the discontinuity across this part of the left-hand cut.

To treat the more distant part of the left-hand cut, we first approximate it in terms of a small number of effective-range parameters by following the procedure described in I and S. This procedure is particularly simple if we first replace all the complex singularities by an equivalent cut on the real axis, with a discontinuity adjusted to give the correct amplitude to the right of the cut.[10] These parameters can, in turn, be calculated if we require that the amplitude and its derivatives be given correctly at some point (or points) $s=s_F$ in the nearby left-hand region. In such a calculation, one must, of course, take into account the right-hand cut, since it, too, contributes to the amplitude. The discontinuity across this cut is, however, determined by the Cutkosky generalized unitarity condition.

In selecting s_F, one must choose as small a value as possible. If it is too large, the more complicated ex-

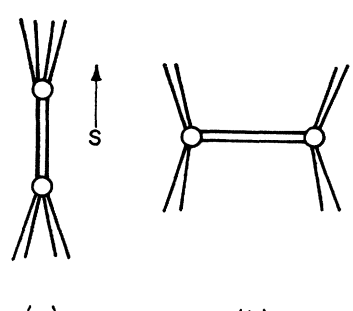

FIG. 2. (a) A typical direct graph; (b) a typical exchange graph.

[6] D. D. Carmony and R. T. Van de Walle, Phys. Rev. Letters 8, 73 (1962).

[7] J. Button, G. R. Kalbfleisch, G. R. Lynch, B. C. Maglić, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. 126, 1858 (1962), to which the reader is also referred for additional references on experimental π-π scattering.

[8] L. D. Landau, Nucl. Phys. 13, 181 (1959).

[9] R. E. Cutkosky, J. Math. Phys. 1, 429 (1960); Phys. Rev. Letters 4, 624 (1960).

[10] For an example of such a replacement, see L. A. P. Balázs, Phys. Rev. 128, 1935 (1962).

change graphs become important in the above calculation. At the same time, the position of s_F should not be too close to the more distant left-hand singularities. Otherwise, a large number of effective-range parameters would be needed to represent these singularities. One must also avoid points which make it necessary to subtract infinity from infinity in the course of the calculation.

The main difficulty associated with the above procedure is that one has, in general, anomalous thresholds in the angle variables, even in the physical region.[11] This prevents the convergence of a partial-wave expansion, which would be needed to obtain the total amplitude by the above method. We shall outline two ways of overcoming this difficulty: (a) We could just calculate the lowest waves by the above approach. Since it is only here that distant singularities (short-range forces) are important,[12] the rest of the amplitude will be given by nearby singularities (long-range forces). These can be handled by more conventional techniques, for instance, the multiple-impulsive peripheral approach proposed by Cutkosky.[13] In this approach, the lower waves are assumed to be given, and the rest of the amplitude is built up from these waves. (b) We could simply proceed with the total amplitude without making a partial-wave expansion. At fixed values of the other variables (for instance, the angle variables), we again have the usual left- and right-hand cuts in the s plane. These may then be treated essentially as they are in a partial-wave amplitude.[14]

Of the two methods, (a) is probably more feasible in practice, at least if numerous simplifying assumptions are made. However, there may be convergence and other difficulties. Method (b), on the other hand, does not seem to have such difficulties, but is probably much more difficult to apply in practice. Whichever method we use, however, we could probably solve the problem only if we explicitly knew all the discontinuities associated with the exchange graphs—or, equivalently, if we knew the amplitudes associated with the vertices of such graphs, since these give the discontinuities through the Cutkosky generalized unitarity condition. In general, we could not solve the problem self-consistently because we have not made use of analyticity in those other variables t_i that are also independent of s. One way of rectifying this situation would be to repeat the above calculation with the total amplitude as a function of each of these other variables. We could then impose the additional condition that the effective-range parameters be the same each time. This should be sufficient to solve the problem.

In the above approach, independent subtractions may have to be made by projecting out one or more of the lowest waves. The value and derivatives of the amplitude at s_F for such waves can, however, always be calculated through crossing and self-consistency.[15] Because of the effective-range approximations used, one may also get divergences in certain integrals which should be convergent. We can remove these by imposing the additional condition that the integrals have, at least approximately, the correct asymptotic behavior. This provides an additional condition for determining the effective-range parameters.

ACKNOWLEDGMENTS

The author wishes to express his deepest gratitude and appreciation to Professor Geoffrey F. Chew for his encouragement and many useful suggestions, to Dr. David Y. Wong for critical comments, and to Dr. Virendra Singh, Dr. Bhalchandra M. Udgaonkar, Michael DerSarkissian, and Alberto Pignotti for many helpful conversations.

APPENDIX: S-WAVE SCATTERING

It can be shown that the P- and D- wave resonances, the dominant features of low-energy π-π scattering, are not affected very much by the S waves. Since the $I=0$, $l=0$ state does have observable effects,[16] however, a calculation of this state would nevertheless be of some interest. For this purpose, we shall use the same effective-range approximation as for the P wave, with $n=2$, $\omega_1=6.25$, and $\omega_2=50$. This time, however, it is more convenient to set $\nu_0=-\omega_1$ in Eq. (3) and $\nu_0=\nu_F=-\frac{2}{3}$ in Eq. (2). We then have effectively only two effective-range parameters, to be determined from the value and derivative of the amplitude at $\nu=\nu_F$. As pointed out in L, Eq. (5) cannot be used to evaluate the latter quantities, since it diverges in this state. But they can be calculated through Eqs. (III.7), (III.15), (III.17), and (III.18) of Chew and Mandelstam,[15] which give

$$A_{(0)0}(\nu_F)\simeq-5\lambda=\tfrac{5}{2}A_2(\nu_F,0), \quad \text{(A1)}$$

$$A_{(0)0}'(\nu_F)\simeq6\nu_F^{-1}A_{(1)1}(\nu_F). \quad \text{(A2)}$$

The quantity $A_{(1)1}(\nu)$ can be taken from the effective-range expression obtained in the self-consistent P-D calculation described earlier, and gives $A_{(0)0}'(\nu_F)=0.719$. The $I=2$ total amplitude $A_2(\nu, \cos\theta)$ can be calculated by using a fixed-ν dispersion relation, which

---

[11] R. E. Cutkosky, Rev. Mod. Phys. **33**, 448 (1961). The anomalous thresholds are associated with long-range forces.

[12] G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin, Inc., New York, 1961).

[13] R. E. Cutosky, Nucl. Phys. (to be published).

[14] This type of approach was first suggested in a somewhat special case by R. Blankenbecler, Phys. Rev. **122**, 983 (1961).

[15] A typical example is the set of exact crossing conditions given by G. F. Chew and S. Mandelstam, Nuovo Cimento **19**, 752 (1961).

[16] A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters **5**, 258 (1960). This experiment has been analyzed by T. N. Truong, *ibid.* **6**, 308 (1961), and by B. Desai, *ibid.* **6**, 497 (1961). The latter author's explanation is essentially equivalent to assuming an $I=0$ S-wave π-π scattering length in the interval (2,3).

gives

$$A_2(\nu_F,0)=\frac{1}{\pi}\int_0^\infty d\nu'\,\mathrm{Im}\tilde{A}_2\left(\nu',1+2\frac{\nu'+1}{\nu_F}\right)\frac{2}{\nu'-\nu_F}. \quad \text{(A3)}$$

We can now insert the P- and D-wave resonances into this expression through Eqs. (10) and (6) with the parameters $\nu_R\Gamma_1^1=4.6$, $\nu_R=5.0$ and $\nu_R^2\Gamma_2^0=4.4$, $\nu_R=9.2$, which were obtained in the coupled P-D calculation. It is found, however, that the $I=0$, $l=0$ state itself cannot be neglected in Eq. (A3). To take it into account, λ—and hence $A_{(0)0}(\nu_F)$—was varied until a calculation of the type described above gave a resonance, which, when added to the P and D resonances in Eq. (A3), gave back the same value of λ through Eq. (A1). Such a self-consistent calculation gives $\lambda=-0.13$.[17] The corresponding values of $F_{(0)0}{}^1+A_{(0)0}(-\omega_1)$ and $F_{(0)0}{}^2$ are -1.50 and 19.0, respectively. These give a scattering length of 3.4, which agrees with the experimental value deduced by Desai.[16]

In the foregoing calculation of $A_2(\nu_F,0)$, the delta-function approximation given by Eqs. (8), (9), and (10) was used for the $I=0$ S-wave resonance. In general, with a large scattering length, such an approximation may be dubious. However, on the basis of some rough estimates, it appears that the approximations (7) and (10) are both reasonable ones in this particular case.

The $I=2$ S-state can be calculated just as any other state, since Eq. (5) does converge here. However, it is somewhat simpler to follow, instead, the same procedure as for $I=0$, but with $A_{(0)2}(\nu_F)\simeq-2\lambda$, and $A_{(0)2}'(\nu_F)\simeq-3\nu_F^{-1}A_{(1)1}(\nu_F)$ instead of Eqs. (A1) and (A2). With $\lambda=-0.13$, this gives $F_{(0)2}{}^1+A_{(0)2}(-\omega_1)=2.08$, and $F_{(0)2}{}^2=-16.2$. The corresponding scattering length is 0.06.[18] At higher energies, the phase shift becomes negative but remains comparatively small.

[17] The fact that the coupling constant λ is not a fundamental constant, but can be calculated through the $I=2$ amplitude was first pointed out by G. F. Chew (private communication). This should be contrasted with the situation in conventional Lagrangian field theory, where λ has to be specified in advance.

[18] This value falls within the experimental limits obtained by J. Kirz, J. Schwartz, and R. D. Tripp, Phys. Rev. 126, 763 (1962).

PHYSICAL REVIEW VOLUME 129, NUMBER 2 15 JANUARY 1963

Elastic μ^- Scattering in Nuclear Emulsion*†

D. Kotelchuck‡, J. G. McEwen§, and J. Orear
Laboratory of Nuclear Studies, Cornell University, I haca, New York

(Received 16 August 1962)

An emulsion stack was exposed to a separated beam of $\sim 2\times 10^7$ μ^- of energy 52±8 MeV at the CERN synchrocyclotron. Muons with endings in a region near the incoming edge of the plates were traced back to scatters, thus discriminating in favor of events with momentum transfers from 100–160 MeV/c. The 78 events found give evidence that the muon behaves merely as a heavy electron, in contradiction to the anomalous muon-nucleus scattering reported in several cosmic-ray experiments. Our data indicate, however, a possibility of a small amount of scattering in excess of that predicted, particularly for momentum transfers >130 MeV/c. This may be ascribed either to unresolvable inelastic scattering, to inaccuracies in the parameters of the nuclear charge distribution, or to the breakdown in the representation of the many-body nucleus by a smoothed-out potential. Of the 78 events, one was an elastic scatter by hydrogen, which is consistent with the Mott scattering formula.

I. INTRODUCTION

In recent years numerous cosmic-ray experiments have indicated an anomalously large nuclear scattering of muons, often giving results consistent with scattering against point nuclei.[1] Difficulties with energy determination of both incoming and scattered muons, pion background, and multiple scattering corrections have indicated a need for similar experiments with accelerator beams of known composition and momentum. Several such experiments have already been performed,[2–7] none of which have given evidence of

* The research in this paper was supported by the National Science Foundation.

† An account of this research has been submitted by D. Kotelchuck in partial fulfillment of the requirements for the degree of Doctor of Philosophy to the Department of Physics, Cornell University.

‡ Present address: Department of Physics, Vanderbilt Universtiy, Nashville, Tennessee.

§ Present address: Department of Physics, University of Southampton, Southampton, England.

[1] For a summary of experiments since 1958, see G. H. Rawitscher, Phys. Rev. 124, 1978 (1961). For experiments before 1958, see G. N. Fowler and A. W. Wolfendale, *Progress in Elementary Particles and Cosmic-Ray Physics* (North-Holland Publishing Company, Amsterdam, 1958), Vol. 4, p. 123.

[2] B. Chidley, G. Hinman, P. Goldstein, R. Summers, and R. Adler, Can J. Phys. 36, 801 (1958).

[3] G. E. Masek, L. D. Heggie, Y. B. Kim, and R. W. Williams, Phys. Rev. 122, 937 (1961).

[4] C. Y. Kim, S. Kaneko, Y. B. Kim, G. E. Masek, and R. W. Williams, Phys. Rev. 122, 1641 (1961).

[5] M. Bardon, P. Franzini, and J. Lee, Phys. Rev. Letters 7, 23 (1961).